Graphs Regularization for Data Sets and Images: Filtering and Semi-Supervised Classification

Vinh Thong Ta, Olivier Lezoray, Abderrahim Elmoataz

Computer Science PhD Thesis
Vision and Image Analysis Group
University of Caen, Low-Normandy (France)

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Outline

1. Weighted Graph Based Regularization Framework

2. Applications in Image and Data Set
   - Filtering
   - Semi-supervised classification

What are the Main Ideas?

From. . .

- Image processing: filtering, denoising
- Data set processing: semi-supervised classification

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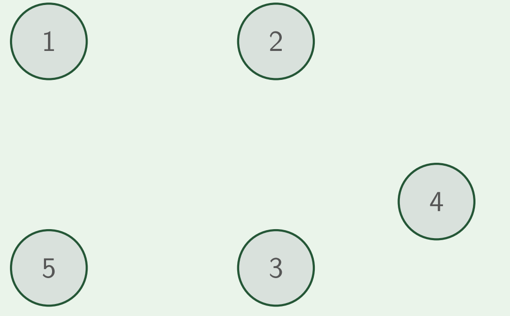
How?

- Weighted graph structure
- Functional regularization based on diffusion processes

Some basis and notations on graphs. . .

What is a Weighted Graph?

$G = (V,E,w)$



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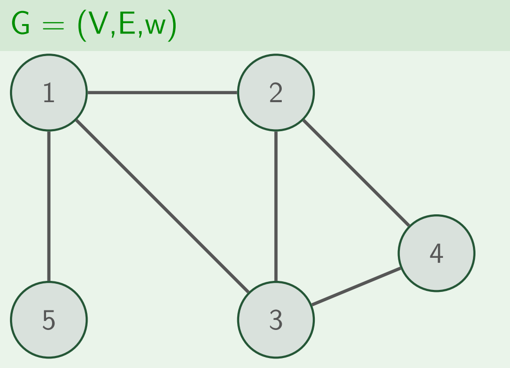

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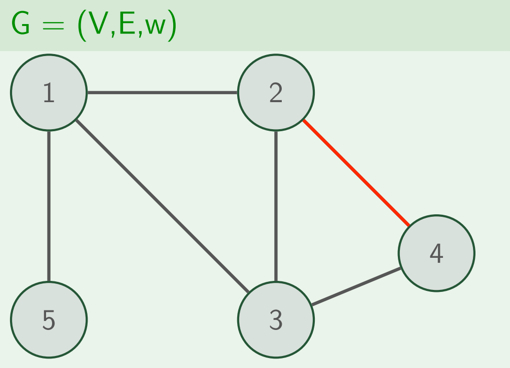

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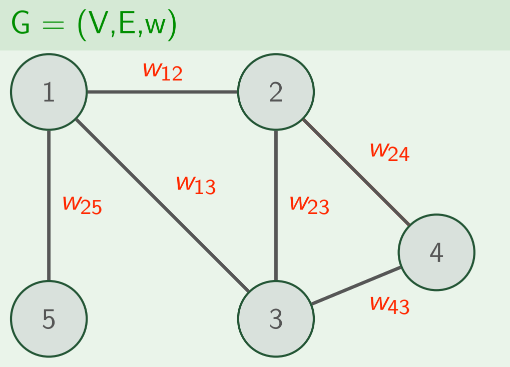

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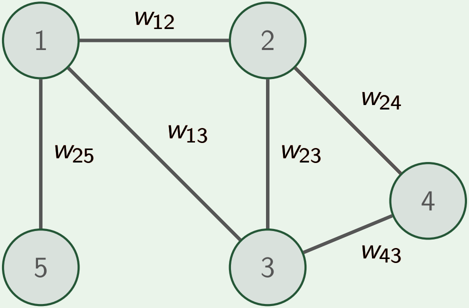
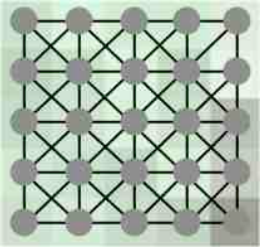


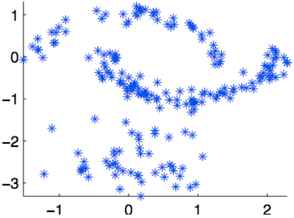
- A finite set of vertices, nodes V
- A subset of edges $E \subseteq V \times V$
- A weight function $w(u,v) : E \rightarrow \mathcal{R}_+$
- G is undirected, connected, no self loop

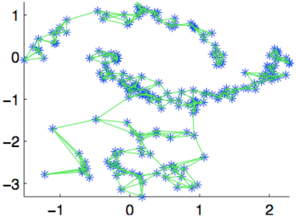
Why Use Graph Representation?

Image:



Data Set:





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Edge Derivative and Difference Operator

$$\partial_v f(u) = \left. \frac{\partial f}{\partial (u,v)} \right|_v = (df)(u, v) = \sqrt{w(u, v)}(f(v) - f(u))$$

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The Norm of the Gradient Operator

$$||\nabla f(v)|| = \sqrt{\sum_{u \sim v} (\partial_v f(u))^2} = \sqrt{\sum_{u \sim v} w(u, v)(f(v) - f(u))^2}$$

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Solutions for $p = 1$ and $p = 2$

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Gauss-Jacobi and $p = 2$ ($\Delta_2 f(v)$: Graph Laplacian Operator)

$$\begin{cases} f^0 = f \\ f^{t+1}(v) = \frac{1}{\lambda + \sum\limits_{u \sim v} w(u,v)} \left(\lambda f^0(v) + \sum\limits_{u \sim v} w(u,v) f^t(u) \right), \quad \forall v \in V. \end{cases}$$

Graph Based Regularization is Not New...

M. Belkin *et al.*
Manifold Regularization: a Geometric Framework for Learning from Examples.
Journal of Machine Learning Research, 2007, to appear.

D. Zhou and B. Schölkopf
Semi-Supervised Learning.
Discrete Regularization, MIT Press, 221–232, 2006.

O. Lezoray and S. Bougleux and A. Elmoataz
Graph Regularization For Color Image Processing.
Computer Vision and Image Understanding 107(1–2): 38–55, 2007.

Application in Filtering. . .

Filtering by Regularization

$G = (V, E, w)$

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- Each vertex is described by a vector of K features

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K independent regularization

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Image Filtering: Classical Example

Corrupted Images

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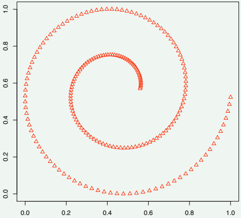
Filtered Images by Regularization, G=8-connectivity grid graph

Data Set Filtering: A Toy Example

Original Data



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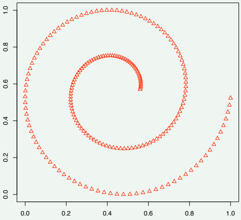

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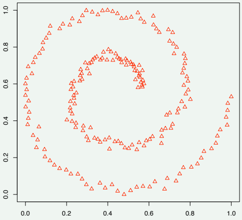

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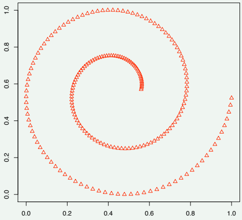

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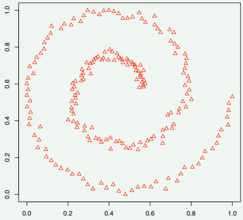
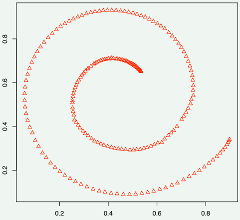


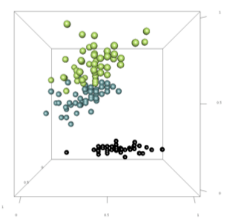
Filtering Result

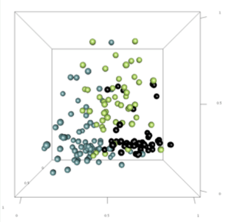


G = Fully connected graph

Data Set Filtering: UCI Data Bases

Original Data

Iris: 

Wine: 

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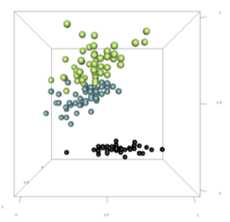
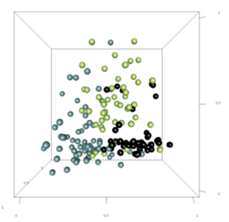
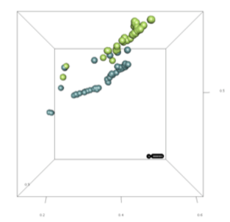
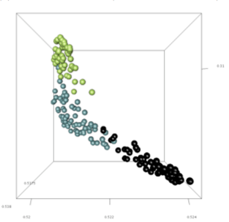
Iris: 

Wine: 

Filtering Results by Regularization

Iris: 

Wine: 

Application in Semi-Supervised Classification. . .

Semi Supervised Classification by Regularization (1)

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- Vertices = Data points
- Classification of K classes problem
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- $\forall i \in [1, K]$:

$$\begin{cases} f_i^0(v) = +1 & \text{if} \quad v \in c_i \quad \text{with } i \in [1, K], \quad \forall v \in C, \\ f_i^0(v) = -1 & \text{otherwise}, \\ f_i^0(v) = 0 & \forall v \in \{V \setminus C\}, \end{cases}$$

Semi Supervised Classification by Regularization (2)

Classification by Regularization: Label Propagation

K independent regularization: $\forall i \in [1, K]$

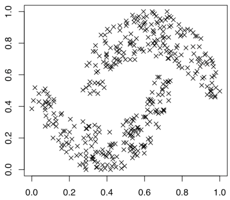
$$f_i^{t+1}(v) = \frac{1}{\lambda + \sum\limits_{u \sim v} w(u,v)} \left(\lambda f_i^0(v) + \sum_{u \sim v} w(u,v) f_i^t(u) \right), \quad \forall v \in V.$$

Decision Function

$$C(v) = argmax \frac{f_i(v)}{\sum\limits_i f_i(v)} \qquad \forall v \in V.$$

The Two Moons Example

Original Data



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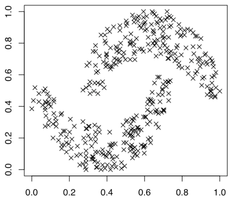
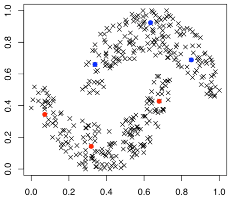


Initial Labels



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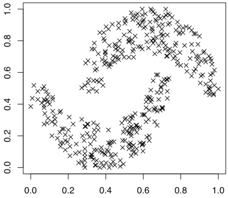

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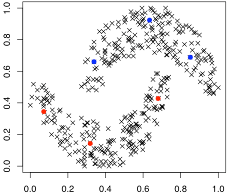

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Classification

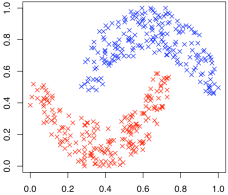
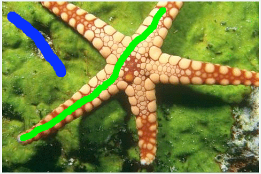
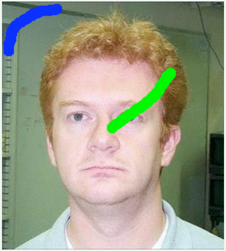


Image Semi Supervised Segmentation (1)

User Marked Images





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Segmentation Results

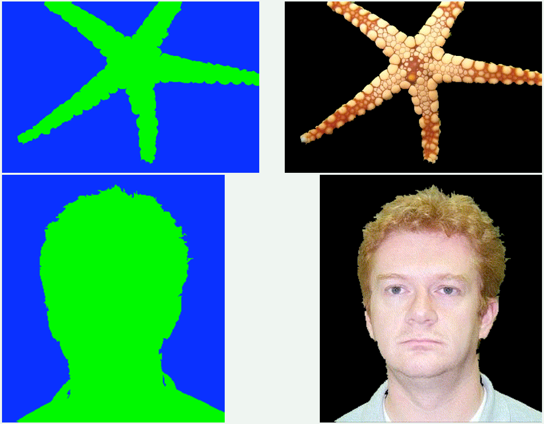

Image Semi Supervised Segmentation (2)

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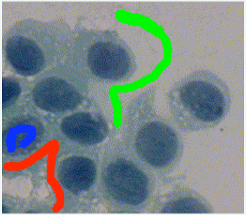

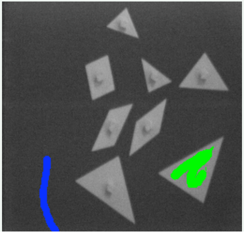

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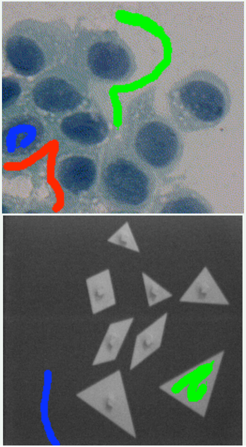

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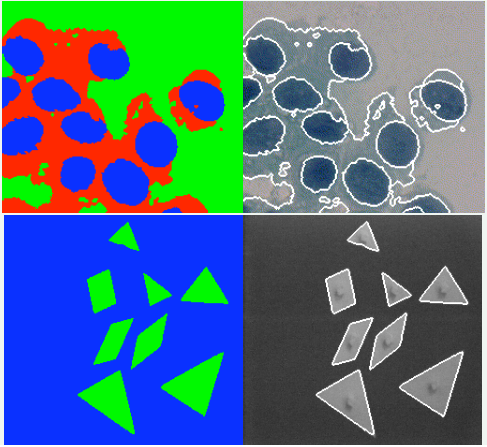

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Future Work

- Demonstrate the benefits of data sets filtering on classification accuracies: A new machine learning pre processing method
- Extends the semi-supervised classification concept for images or objects categorization